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4037/12

October/November 2013

2 hours

No Additional Materials are required.

DO **NOT** WRITE IN ANY BARCODES.

You are reminded of the need for clear presentation in your answers.

The total number of marks for this paper is 80.

This document consists of **16** printed pages.

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{r} a^{n-r} b^r + \dots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}.$

2. TRIGONOMETRY*Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

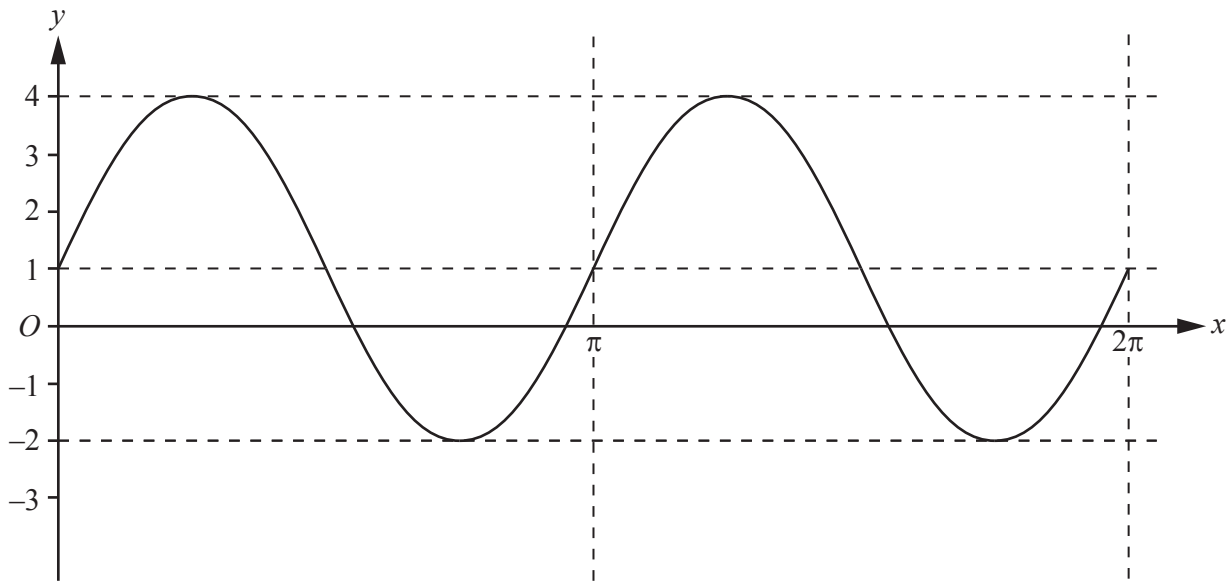
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

- 1 The diagram shows the graph of $y = a \sin(bx) + c$ for $0 \leq x \leq 2\pi$, where a , b and c are positive integers.

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State the value of a , of b and of c .

[3]

$a =$

$b =$

$c =$

- 2 Find the set of values of k for which the curve $y = (k + 1)x^2 - 3x + (k + 1)$ lies below the x -axis.

[4]

3 Show that $\frac{1 + \sin \theta}{\cos \theta} + \frac{\cos \theta}{1 + \sin \theta} = 2 \sec \theta$.

[4]

*For
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4 The sets A and B are such that

$$A = \left\{x: \cos x = \frac{1}{2}, 0^\circ \leq x \leq 620^\circ\right\},$$

$$B = \{x: \tan x = \sqrt{3}, 0^\circ \leq x \leq 620^\circ\}.$$

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(i) Find $n(A)$. [1]

(ii) Find $n(B)$. [1]

(iii) Find the elements of $A \cup B$. [1]

(iv) Find the elements of $A \cap B$. [1]

5 (i) Find $\int (9 + \sin 3x) \mathrm{d}x$.

[3]

*For
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Use*

(ii) Hence show that $\int_{\frac{\pi}{9}}^{\pi} (9 + \sin 3x) \mathrm{d}x = a\pi + b$, where a and b are constants to be found. [3]

- 6 The function $f(x) = ax^3 + 4x^2 + bx - 2$, where a and b are constants, is such that $2x - 1$ is a factor. Given that the remainder when $f(x)$ is divided by $x - 2$ is twice the remainder when $f(x)$ is divided by $x + 1$, find the value of a and of b . [6]

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Use*

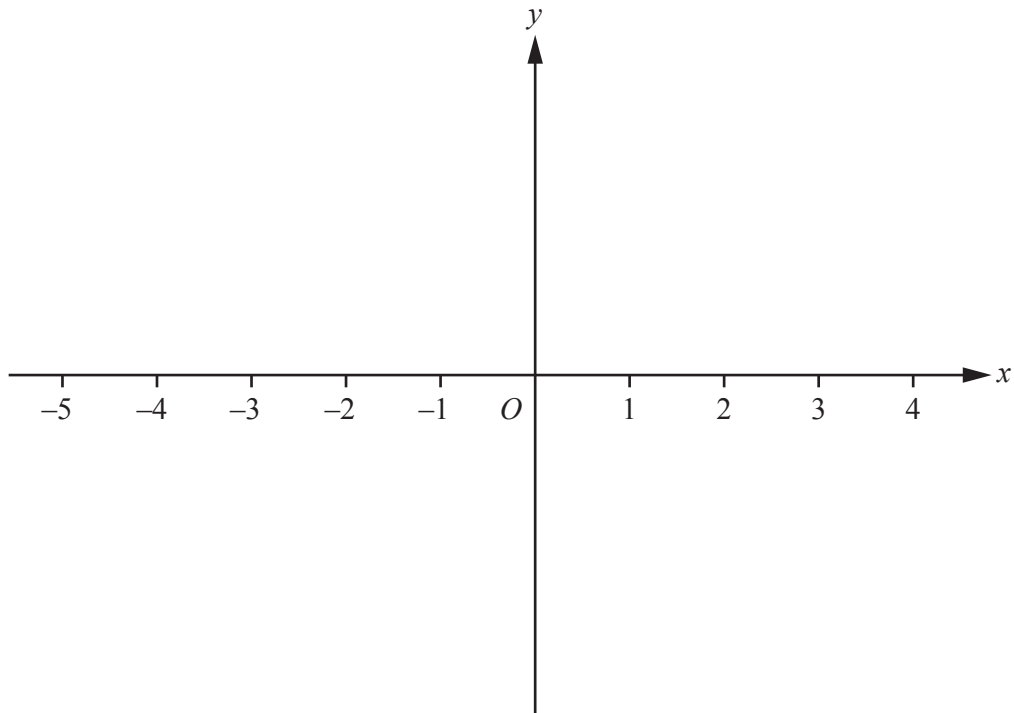
- 7 (a) (i) Find how many different 4-digit numbers can be formed from the digits 1, 3, 5, 6, 8 and 9 if each digit may be used only once. [1]
- (ii) Find how many of these 4-digit numbers are even. [1]
- (b) A team of 6 people is to be selected from 8 men and 4 women. Find the number of different teams that can be selected if
- (i) there are no restrictions, [1]
- (ii) the team contains all 4 women, [1]
- (iii) the team contains at least 4 men. [3]

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Use*

- 8 (i) On the grid below, sketch the graph of $y = |(x - 2)(x + 3)|$ for $-5 \leq x \leq 4$, and state the coordinates of the points where the curve meets the coordinate axes.

[4]

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- (ii) Find the coordinates of the stationary point on the curve $y = |(x - 2)(x + 3)|$. [2]

- (iii) Given that k is a positive constant, state the set of values of k for which $|(x - 2)(x + 3)| = k$ has 2 solutions only. [1]

- 9 (a) Differentiate $4x^3 \ln(2x + 1)$ with respect to x .

[3]

*For
Examiner's
Use*

- (b) (i) Given that $y = \frac{2x}{\sqrt{x+2}}$, show that $\frac{dy}{dx} = \frac{x+4}{(\sqrt{x+2})^3}$.

[4]

(ii) Hence find $\int \frac{5x + 20}{(\sqrt{x + 2})^3} dx$.

[2]

*For
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Use*

(iii) Hence evaluate $\int_2^7 \frac{5x + 20}{(\sqrt{x + 2})^3} dx$.

[2]

10 Solutions to this question by accurate drawing will not be accepted.*For
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The points $A(-3, 2)$ and $B(1, 4)$ are vertices of an isosceles triangle ABC , where angle $B = 90^\circ$.

(i) Find the length of the line AB . [1]

(ii) Find the equation of the line BC . [3]

- (iii) Find the coordinates of each of the two possible positions of C .

[6]

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11 (a) It is given that the matrix $\mathbf{A} = \begin{pmatrix} 2 & 3 \\ 4 & 1 \end{pmatrix}$.

(i) Find $\mathbf{A} + 2\mathbf{I}$.

[1]

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(ii) Find $\mathbf{A}^2$.

[2]

(iii) Using your answer to part **(ii)** find the matrix $\mathbf{B}$ such that $\mathbf{A}^2\mathbf{B} = \mathbf{I}$.

[2]

- (b) Given that the matrix $\mathbf{C} = \begin{pmatrix} x & -1 \\ x^2 - x + 1 & x - 1 \end{pmatrix}$, show that $\det \mathbf{C} \neq 0$.

[4]

*For
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-
- 12 (a) A function f is such that $f(x) = 3x^2 - 1$ for $-10 \leq x \leq 8$.

- (i) Find the range of f .

[3]

- (ii) Write down a suitable domain for f for which f^{-1} exists.

[1]

Question 12(b) is printed on the next page.

(b) Functions g and h are defined by

$$g(x) = 4e^x - 2 \text{ for } x \in \mathbb{R},$$

$$h(x) = \ln 5x \text{ for } x > 0.$$

(i) Find $g^{-1}(x)$.

[2]

(ii) Solve $gh(x) = 18$.

[3]

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