Cambridge O Level

Cambridge International Examinations
Cambridge Ordinary Level

CANDIDATE NAME	

CENTRE NUMBER						CANDIDATE NUMBER				

3621984096

ADDITIONAL MATHEMATICS **4037/23**

Paper 2 **October/November 2017**

2 hours

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This document consists of **16** printed pages.

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CAMBRIDGE International Examinations

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}\, b + \binom{n}{2}a^{n-2}\, b^2 + \ldots + \binom{n}{r}a^{n-r}\, b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}\, bc \sin A$$

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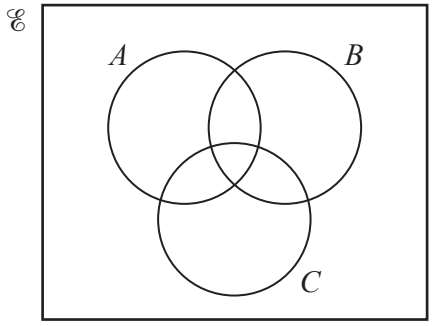
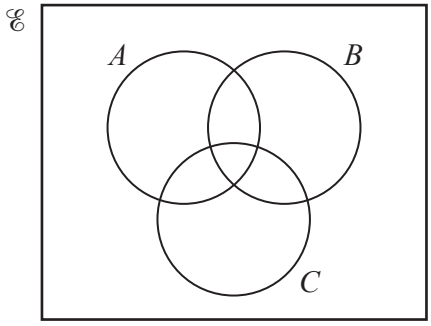
1 **(a)** On each of the diagrams below, shade the region which represents the given set.

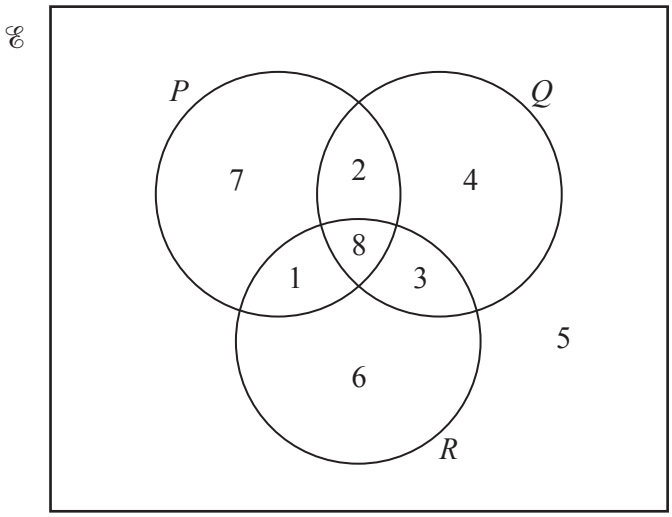


$(A \cup B) \cap C'$



$(A \cap B') \cup C$ [2]

(b)



The Venn diagram shows the number of elements in each of its subsets.

Complete the following.

n(P') = ...

n(($Q \cup R$) $\cap$ P) = ...

n($Q' \cup P$) = ... [3]

2 Solve the equation $|3x - 1| = |5 + x|$. [3]

3 Find integers p and q such that $\dfrac{p}{\sqrt{3} - 1} + \dfrac{1}{\sqrt{3} + 1} = q + 3\sqrt{3}$. [4]

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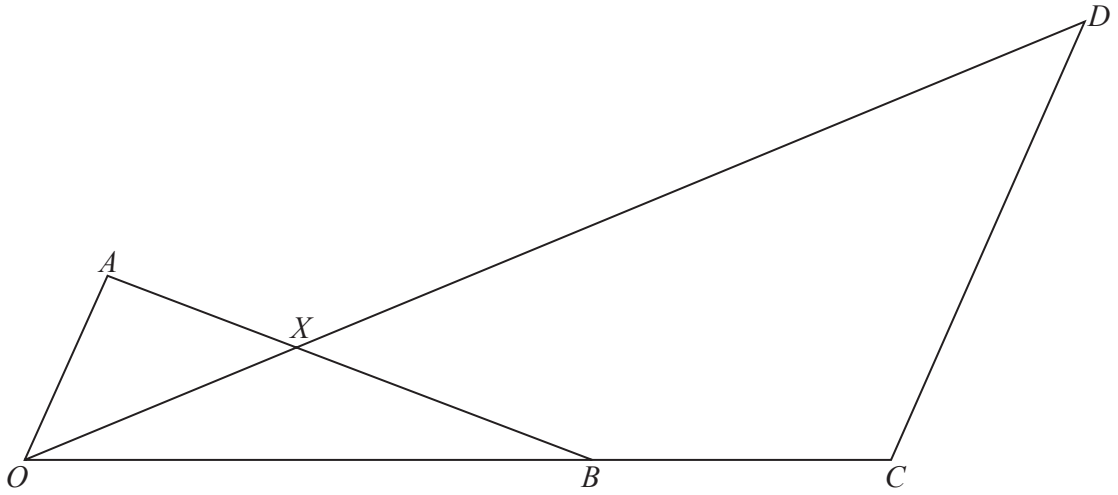
4 Solve the simultaneous equations

$$\log_3(x+1) = 1 + \log_3 y,$$

$$\log_3(x-y) = 2.$$ [5]

5



The diagram shows points O, A, B, C, D and X. The position vectors of A, B and C relative to O are $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$ and $\overrightarrow{OC} = \frac{3}{2}\mathbf{b}$. The vector $\overrightarrow{CD} = 3\mathbf{a}$.

(i) If $\overrightarrow{OX} = \lambda\overrightarrow{OD}$ express $\overrightarrow{OX}$ in terms of λ, $\mathbf{a}$ and $\mathbf{b}$. [1]

(ii) If $\overrightarrow{AX} = \mu\overrightarrow{AB}$ express $\overrightarrow{OX}$ in terms of μ, $\mathbf{a}$ and $\mathbf{b}$. [2]

(iii) Use your two expressions for $\overrightarrow{OX}$ to find the value of λ and of μ. [3]

(iv) Find the ratio $\frac{AX}{XB}$. [1]

(v) Find the ratio $\frac{OX}{XD}$. [1]

6 The functions f and g are defined for real values of x by

$$\mathrm{f}(x) = (x+2)^2 + 1,$$

$$\mathrm{g}(x) = \frac{x-2}{2x-1}, \; x \neq \frac{1}{2}.$$

(i) Find $\mathrm{f}^2(-3)$. [2]

(ii) Show that $\mathrm{g}^{-1}(x) = \mathrm{g}(x)$. [3]

(iii) Solve $\mathrm{gf}(x) = \frac{8}{19}$. [4]

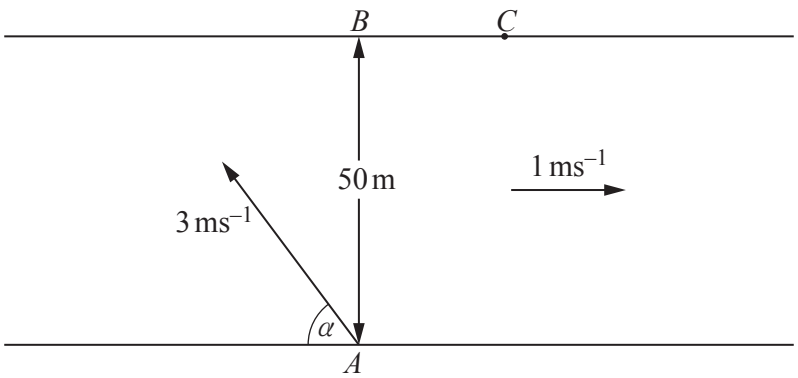
7 A particle moving in a straight line passes through a fixed point O. Its velocity, v ms^{-1}, t s after passing through O, is given by $v = 3\cos 2t - 1$ for $t \geqslant 0$.

(i) Find the value of t when the particle is first at rest. [2]

(ii) Find the displacement from O of the particle when $t = \frac{\pi}{4}$. [3]

(iii) Find the acceleration of the particle when it is first at rest. [3]

8



A man, who can row a boat at $3\,\text{ms}^{-1}$ in still water, wants to cross a river from A to B as shown in the diagram. AB is perpendicular to both banks of the river. The river, which is 50 m wide, is flowing at $1\,\text{ms}^{-1}$ in the direction shown. The man points his boat at an angle $\alpha°$ to the bank. Find

(i) the angle α, [2]

(ii) the resultant speed of the boat from A to B, [2]

(iii) the time taken for the boat to travel from A to B. [2]

On another occasion the man points the boat in the same direction but the river speed has increased to $1.8\,\text{ms}^{-1}$ and as a result he lands at the point C.

(iv) State the time taken for the boat to travel from A to C and hence find the distance BC. [2]

9 **(i)** Show that $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\ln x}{x^3}\right) = \frac{1 - 3\ln x}{x^4}$. [3]

(ii) Find the exact coordinates of the stationary point of the curve $y = \frac{\ln x}{x^3}$. [3]

(iii) Use the result from part **(i)** to find $\int \left(\frac{\ln x}{x^4} \right) \mathrm{d}x$. [4]

10 **(a)** Show that $\dfrac{\sin x}{1+\cos x}+\dfrac{1+\cos x}{\sin x}=2\operatorname{cosec} x$. [3]

(b) Solve the following equations.

(i) $\cot^2 y+\operatorname{cosec} y-5=0$ for $0° \leqslant y \leqslant 360°$ [5]

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(ii) $\cos\left(2z+\frac{\pi}{4}\right)=-\frac{\sqrt{3}}{2}$ for $0 \leqslant z \leqslant \pi$ radians [4]

Question 11 is printed on the next page.

11 The cubic equation $x^3 + ax^2 + bx - 36 = 0$ has a repeated positive integer root.

(i) If the repeated root is $x = 3$ find the other positive root and the value of a and of b. [4]

(ii) There are other possible values of a and b for which the cubic equation has a repeated positive integer root. In each case state all three integer roots of the equation. [4]

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