Cambridge O Level

**Cambridge International Examinations**
Cambridge Ordinary Level

| CANDIDATE NAME | |
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| CENTRE NUMBER | | | | | | CANDIDATE NUMBER | | | | |
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*3679564830*

**ADDITIONAL MATHEMATICS** **4037/12**

Paper 1 **May/June 2018**

**2 hours**

Candidates answer on the Question Paper.

No Additional Materials are required.

**READ THESE INSTRUCTIONS FIRST**

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This document consists of **16** printed pages.

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CAMBRIDGE International Examinations

***Mathematical Formulae***

## 1. ALGEBRA

*Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

*Binomial Theorem*

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}\, b + \binom{n}{2}a^{n-2}\, b^2 + \ldots + \binom{n}{r}a^{n-r}\, b^r + \ldots + b^n,$$

where $n$ is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

## 2. TRIGONOMETRY

*Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

*Formulae for $\Delta ABC$*

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}\, bc \sin A$$

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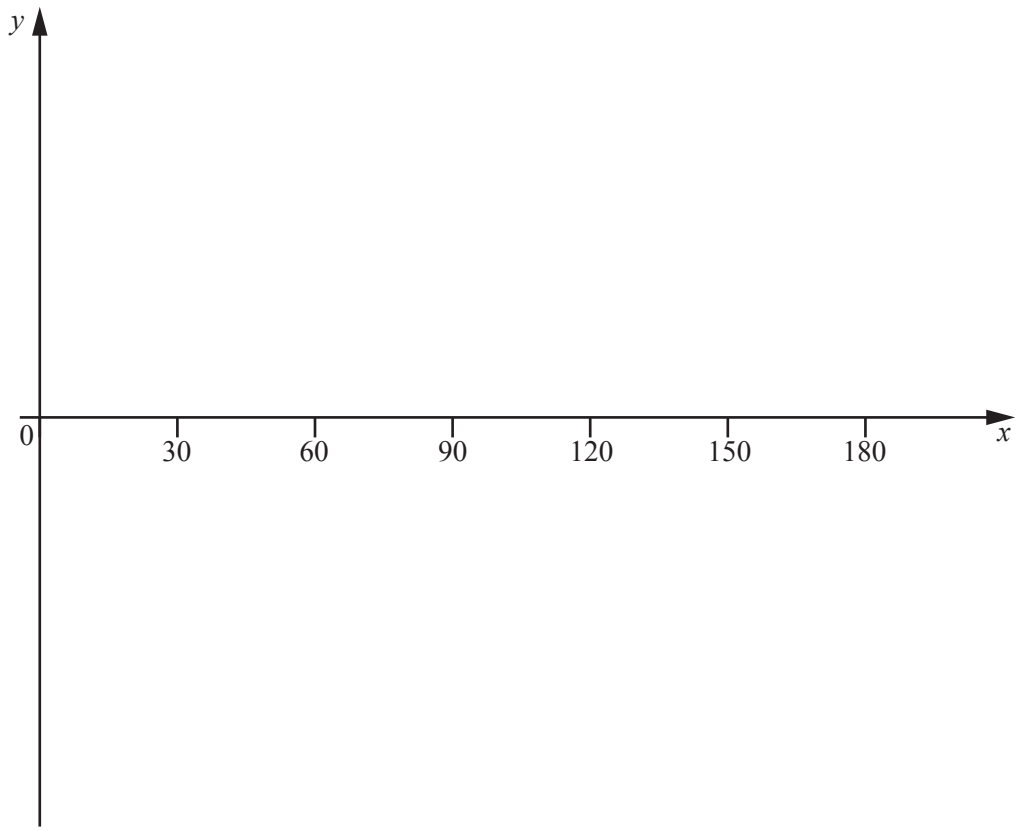
**1** It is given that $y = 1 + \tan 3x$.

**(i)** State the period of $y$. [1]

**(ii)** On the axes below, sketch the graph of $y = 1 + \tan 3x$ for $0° \leqslant x° \leqslant 180°$. [3]



**2** Find the values of $k$ for which the line $y = 1 - 2kx$ does not meet the curve $y = 9x^2 - (3k + 1)x + 5$. [5]

**3** The variables $x$ and $y$ are such that when $e^y$ is plotted against $x^2$, a straight line graph passing through the points (5, 3) and (3, 1) is obtained. Find $y$ in terms of $x$. [5]

**4** A particle $P$ moves so that its displacement, $x$ metres from a fixed point $O$, at time $t$ seconds, is given by $x = \ln(5t + 3)$.

**(i)** Find the value of $t$ when the displacement of $P$ is 3m. [2]

**(ii)** Find the velocity of $P$ when $t = 0$. [2]

**(iii)** Explain why, after passing through $O$, the velocity of $P$ is never negative. [1]

**(iv)** Find the acceleration of $P$ when $t = 0$. [2]

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**5** **(i)** The first three terms in the expansion of $\left(3 - \frac{1}{9x}\right)^5$ can be written as $a + \frac{b}{x} + \frac{c}{x^2}$. Find the value of each of the constants $a$, $b$ and $c$. [3]

**(ii)** Use your values of $a$, $b$ and $c$ to find the term independent of $x$ in the expansion of

$$\left(3 - \frac{1}{9x}\right)^5 (2 + 9x)^2.$$ [3]

**6** Find the coordinates of the stationary point of the curve $y = \dfrac{x+2}{\sqrt{2x-1}}$. [6]

**7** A population, $B$, of a particular bacterium, $t$ hours after measurements began, is given by $B = 1000e^{\frac{t}{4}}$.

**(i)** Find the value of $B$ when $t = 0$. [1]

**(ii)** Find the time taken for $B$ to double in size. [3]

**(iii)** Find the value of $B$ when $t = 8$. [1]

**8** **(a)** Solve $3\cos^2\theta + 4\sin\theta = 4$ for $0° \leqslant \theta \leqslant 180°$. [4]

**(b)** Solve $\sin 2\phi = \sqrt{3}\cos 2\phi$ for $-\frac{\pi}{2} \leqslant \phi \leqslant \frac{\pi}{2}$ radians. [4]

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**9** **(a)** **(i)** Solve $\lg x = 3$. [1]

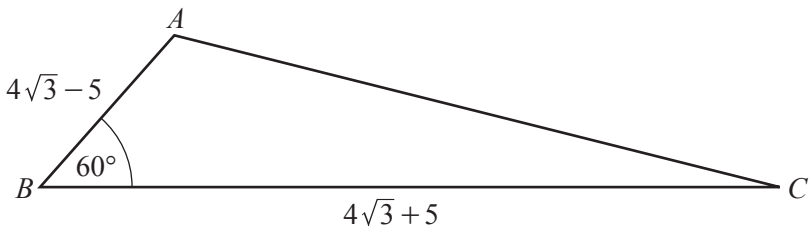
**(ii)** Write $\lg a - 2\lg b + 3$ as a single logarithm. [3]

**(b)** **(i)** Solve $x - 5 + \frac{6}{x} = 0$. [2]

**(ii)** Hence, showing all your working, find the values of $a$ such that $\log_4 a - 5 + 6\log_a 4 = 0$. [3]

**10 Do not use a calculator in this question.**

All lengths in this question are in centimetres.



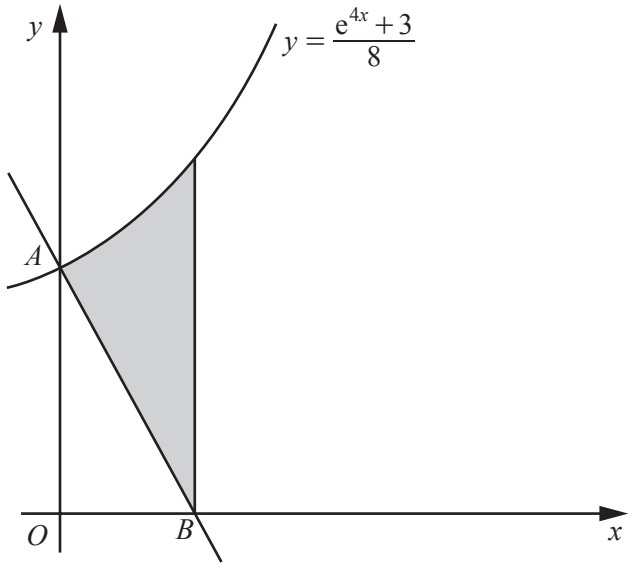

The diagram shows the triangle $ABC$, where $AB = 4\sqrt{3} - 5$, $BC = 4\sqrt{3} + 5$ and angle $ABC = 60°$.

It is known that $\sin 60° = \frac{\sqrt{3}}{2}$, $\cos 60° = \frac{1}{2}$, $\tan 60° = \sqrt{3}$.

**(i)** Find the exact value of $AC$. [4]

**(ii)** Hence show that $\operatorname{cosec} ACB = \dfrac{2\sqrt{p}}{q}(4\sqrt{3}+5)$, where $p$ and $q$ are integers. [4]

**11**




The diagram shows the graph of the curve $y = \frac{e^{4x}+3}{8}$. The curve meets the $y$-axis at the point $A$.

The normal to the curve at $A$ meets the $x$-axis at the point $B$. Find the area of the shaded region enclosed by the curve, the line $AB$ and the line through $B$ parallel to the $y$-axis. Give your answer in the form $\frac{e}{a}$, where $a$ is a constant. You must show all your working.

[10]

**Question 12 is printed on the next page.**

**12 Do not use a calculator in this question.**

**(a)** Given that $\dfrac{6^p \times 8^{p+2} \times 3^q}{9^{2q-3}}$ is equal to $2^7 \times 3^4$, find the value of each of the constants $p$ and $q$. [3]

**(b)** Using the substitution $u = x^{\frac{1}{3}}$, or otherwise, solve $4x^{\frac{1}{3}} + x^{\frac{2}{3}} + 3 = 0$. [4]

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