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4037/22

October/November 2019

2 hours

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

DO **NOT** WRITE IN ANY BARCODES.

You are reminded of the need for clear presentation in your answers.

The total number of marks for this paper is 80.

This document consists of **16** printed pages.

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{r} a^{n-r} b^r + \dots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

2. TRIGONOMETRY*Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

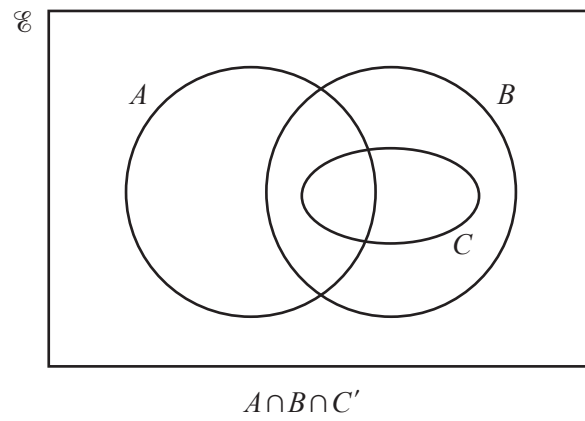
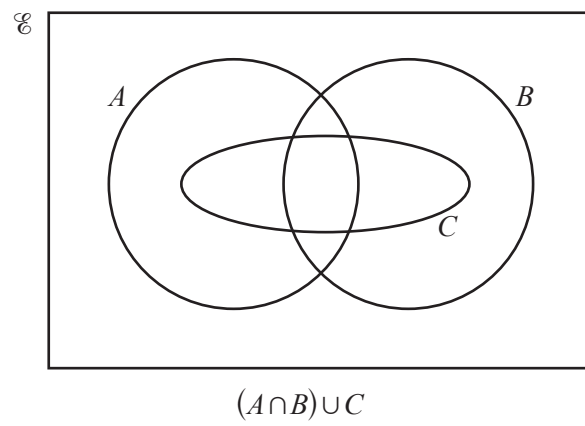
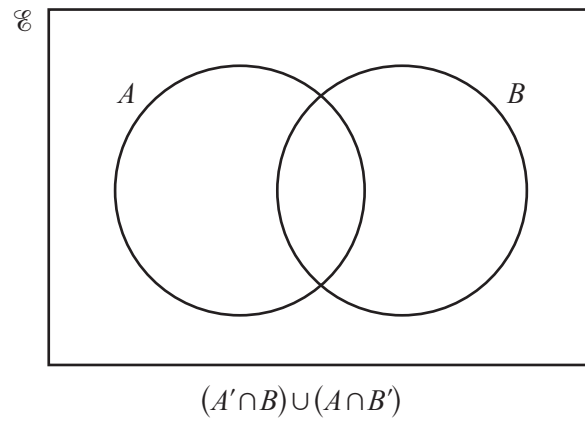
Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

- 1 On each of the Venn diagrams below, shade the region indicated.



[3]

- 2 Given that $y = 2 \sin 3x + \cos 3x$, show that $\frac{d^2y}{dx^2} + \frac{dy}{dx} + 3y = k \sin 3x$, where k is a constant to be determined. [5]

- 3 A 5-digit code is formed using the following characters.

Letters	a	e	i	o	u	
Numbers	1	2	3	4	5	6
Symbols	@	*	#			

No character can be repeated in a code. Find the number of possible codes if

- (i) there are no restrictions, [2]

- (ii) the code starts with a symbol followed by two letters and then two numbers, [2]

- (iii) the first two characters are numbers, and no other numbers appear in the code. [2]

- 4 Find the values of k for which the line $y = kx + 3$ does not meet the curve $y = x^2 + 5x + 12$. [5]

5 At the point where $x = 1$ on the curve $y = \frac{k}{(x+1)^2}$, the normal has a gradient of $\frac{1}{3}$.

(i) Find the value of the constant k . [4]

(ii) Using your value of k , find the equation of the tangent to the curve at $x = 2$. [3]

6 (i) Show that $\frac{\tan x}{1 + \sec x} + \frac{1 + \sec x}{\tan x} = \frac{2}{\sin x}$.

[5]

- (ii) Hence solve the equation $\frac{\tan x}{1 + \sec x} + \frac{1 + \sec x}{\tan x} = 1 + 3 \sin x$ for $0^\circ \leq x \leq 180^\circ$. [4]

- 7 (a) The cubic equation $x^3 + ax^2 + bx - 40 = 0$ has three positive integer roots. Two of the roots are 2 and 4. Find the other root and the value of each of the integers a and b . [4]

(b) Do not use a calculator in this question.

Solve the equation $x^3 - 5x^2 - 46x - 40 = 0$ given that it has three integer roots, only one of which is positive. [4]

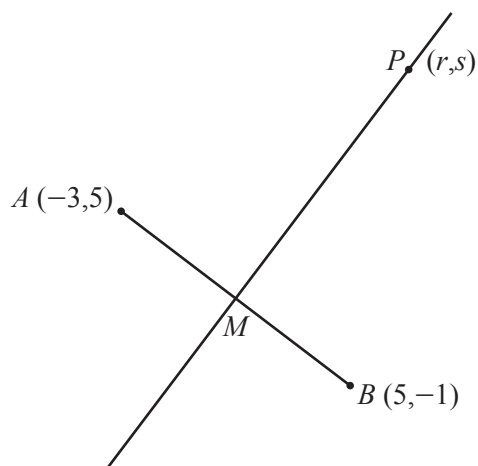
8 (i) A particle A travels with a speed of 6.5 ms^{-1} in the direction $-5\mathbf{i} - 12\mathbf{j}$. Find the velocity, $\mathbf{v}_A$, of A . [2]

(ii) A particle B travels with velocity $\mathbf{v}_B = 12\mathbf{i} - 9\mathbf{j}$. Find the speed, in ms^{-1} , of B . [2]

Particle A starts moving from the point with position vector $20\mathbf{i} - 7\mathbf{j}$. At the same time particle B starts moving from the point with position vector $-67\mathbf{i} + 11\mathbf{j}$.

(iii) Find $\mathbf{r}_A$, the position vector of A after t seconds, and $\mathbf{r}_B$, the position vector of B after t seconds. [2]

(iv) Find the time when the particles collide and the position vector of the point of collision. [3]



The diagram shows the points $A(-3, 5)$ and $B(5, -1)$. The mid-point of AB is M and the line PM is perpendicular to AB . The point P has coordinates (r, s) .

- (i) Find the equation of the line PM in the form $y = mx + c$, where m and c are exact constants. [5]

- (ii) Hence find an expression for s in terms of r . [1]

- (iii) Given that the length of PM is 10 units, find the value of r and of s .

[5]

10 (i) Given that $y = \frac{\ln x}{x^2}$, find $\frac{dy}{dx}$. [3]

(ii) Find the coordinates of the stationary point on the curve $y = \frac{\ln x}{x^2}$. [3]

(iii) Using your answer to **part (i)**, find $\int \frac{\ln x}{x^3} dx$. [3]

(iv) Hence evaluate $\int_1^2 \frac{\ln x}{x^3} dx$. [2]

Question 11 is printed on the next page.

11 Do not use a calculator in this question.

Solve the quadratic equation $(\sqrt{5} - 3)x^2 + 3x + (\sqrt{5} + 3) = 0$, giving your answers in the form $a + b\sqrt{5}$, where a and b are constants. [6]

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