Cambridge Assessment International Education

Cambridge O Level

CANDIDATE NAME	

CENTRE NUMBER						CANDIDATE NUMBER				

0765613034

ADDITIONAL MATHEMATICS **4037/12**

Paper 1 **May/June 2021**

2 hours

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages.

DC (LO) 214475
© UCLES 2021

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $\quad u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $\quad u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

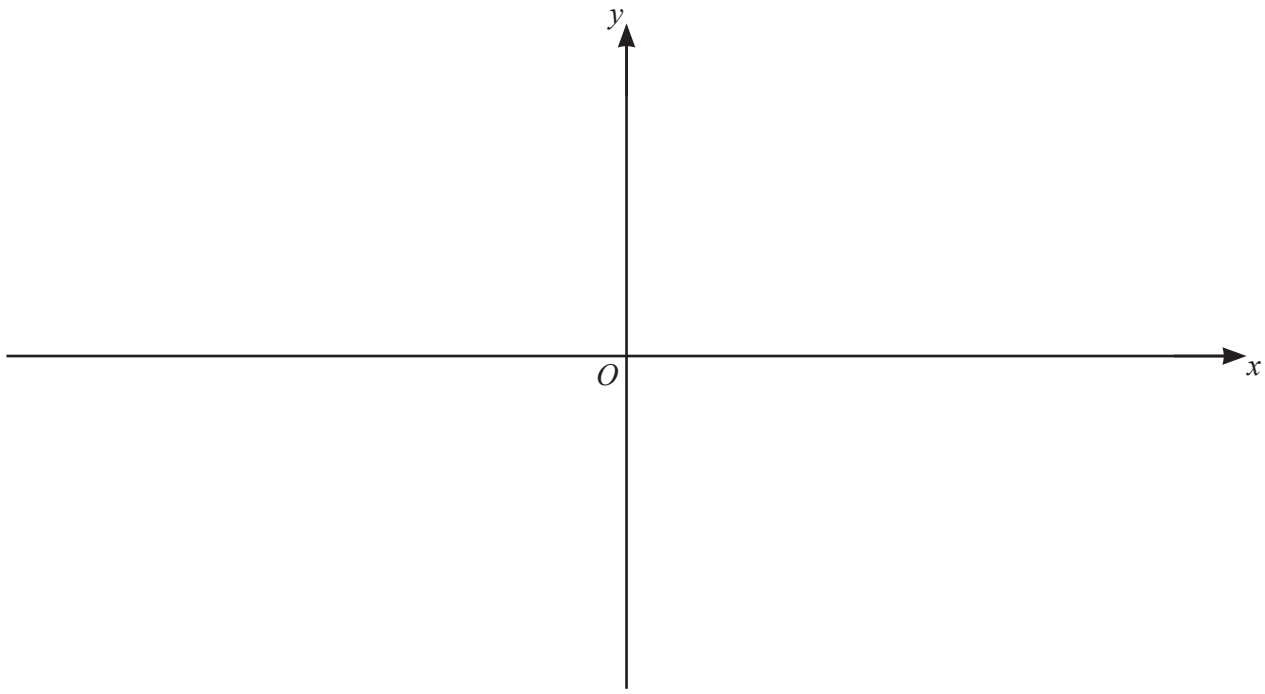
$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

© UCLES 2021

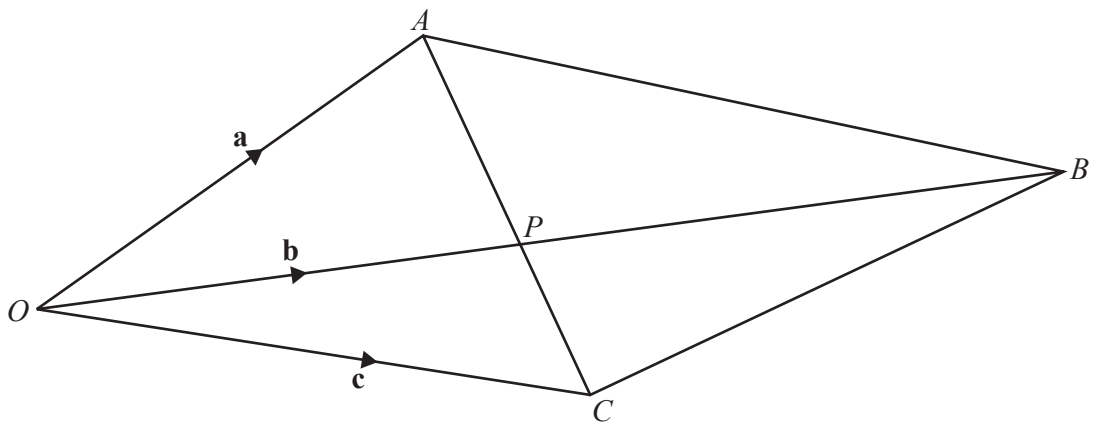
1 Write $\dfrac{(pqr)^{-2}r^{\frac{1}{3}}}{\left(p^2r\right)^{-1}q^3}$ in the form $p^aq^br^c$, where a, b and c are constants. [3]

2 **(a)** On the axes, sketch the graph of $y = |4 - 3x|$, stating the intercepts with the coordinate axes. [2]



(b) Solve the inequality $|4 - 3x| \geqslant 7$. [3]

3



The diagram shows the quadrilateral $OABC$ such that $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$ and $\overrightarrow{OC} = \mathbf{c}$. The lines OB and AC intersect at the point P, such that $AP : PC = 3 : 2$.

(a) Find $\overrightarrow{OP}$ in terms of **a** and **c**. [3]

(b) Given also that $OP : PB = 2 : 3$, show that $2\mathbf{b} = 3\mathbf{c} + 2\mathbf{a}$. [2]

4 A curve is such that $\frac{d^2y}{dx^2} = (3x+2)^{-\frac{1}{3}}$. The curve has gradient 4 at the point $(2, 6.2)$. Find the equation of the curve. [6]

5 **(a)** Given that $\log_a p + \log_a 5 - \log_a 4 = \log_a 20$, find the value of p. [2]

(b) Solve the equation $3^{2x+1} + 8(3^x) - 3 = 0$. [3]

(c) Solve the equation $4\log_y 2 + \log_2 y = 4$. [3]

6 DO NOT USE A CALCULATOR IN THIS QUESTION.

A curve has equation $y = \left(3 + \sqrt{5}\right)x^2 - 8\sqrt{5}x + 60$.

(a) Find the x-coordinate of the stationary point on the curve, giving your answer in the form $a + b\sqrt{5}$, where a and b are integers. [4]

(b) Hence find the y-coordinate of this stationary point, giving your answer in the form $c\sqrt{5}$, where c is an integer. [3]

7 **(a)** A six-character password is to be made from the following eight characters.

Digits	1	3	5	8	9
Symbols	*	$	#		

No character may be used more than once in a password.

Find the number of different passwords that can be chosen if

(i) there are no restrictions, [1]

(ii) the password starts with a digit and finishes with a digit, [2]

(iii) the password starts with three symbols. [2]

(b) The number of combinations of 5 objects selected from n objects is six times the number of combinations of 4 objects selected from $n - 1$ objects. Find the value of n. [3]

8 Variables x and y are such that $y = Ax^b$, where A and b are constants. When $\lg y$ is plotted against $\lg x$, a straight line graph passing through the points (0.61, 0.57) and (5.36, 4.37) is obtained.

(a) Find the value of A and of b. [5]

Using your values of A and b, find

(b) the value of y when $x = 3$, [2]

(c) the value of x when $y = 3$. [2]

9 **(a)** The first three terms of an arithmetic progression are $-4, 8, 20$. Find the smallest number of terms for which the sum of this arithmetic progression is greater than 2000. [4]

(b) The 7th and 9th terms of a geometric progression are 27 and 243 respectively. Given that the geometric progression has a positive common ratio, find

(i) this common ratio, [2]

(ii) the 30th term, giving your answer as a power of 3. [2]

(c) Explain why the geometric progression $1,\ \sin\theta,\ \sin^2\theta,\ \ldots$ for $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$, where θ is in radians, has a sum to infinity. [2]

10 **(a)** Solve the equation $\sin\alpha\,\mathrm{cosec}^2\alpha + \cos\alpha\sec^2\alpha = 0$ for $-\pi < \alpha < \pi$, where α is in radians. [4]

(b) **(i)** Show that $\dfrac{\cos\theta}{1-\sin\theta}+\dfrac{1-\sin\theta}{\cos\theta}=2\sec\theta$. [4]

(ii) Hence solve the equation $\dfrac{\cos 3\phi}{1-\sin 3\phi}+\dfrac{1-\sin 3\phi}{\cos 3\phi}=4$ for $0° \leqslant \phi \leqslant 180°$. [4]

Question 11 is printed on the next page.

11 The normal to the curve $y = \dfrac{\ln\left(x^2+2\right)}{2x-3}$ at the point where $x = 2$ meets the y-axis at the point P.

Find the coordinates of P. [7]

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge Assessment International Education Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cambridgeinternational.org after the live examination series.

Cambridge Assessment International Education is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which itself is a department of the University of Cambridge.

© UCLES 2021