



CANDIDATE
NAME

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CENTRE
NUMBER

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CANDIDATE
NUMBER

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4037/21

May/June 2023

2 hours

You must answer on the question paper.

No additional materials are needed.

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages.

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY*Identities*

$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

Formulae for $\triangle ABC$

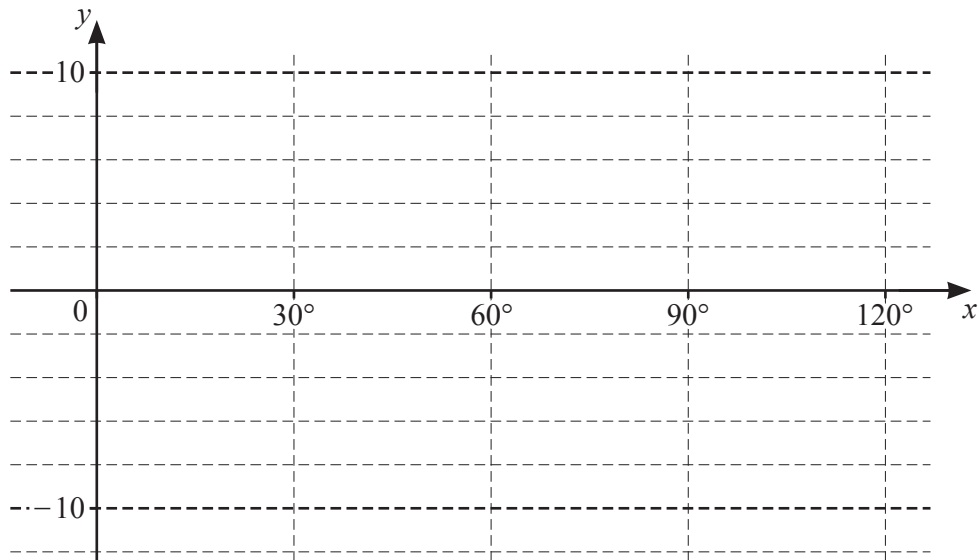
$$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ a^2 &= b^2 + c^2 - 2bc \cos A \\ \Delta &= \frac{1}{2}bc \sin A\end{aligned}$$

- 1 Variables x and y are such that when $\lg y$ is plotted against $\sqrt{x}$ a straight line passing through the points $(1, 5)$ and $(2.5, 8)$ is obtained. Show that $y = A \times b^{\sqrt{x}}$ where A and b are constants to be found. [4]

2 The function g is defined for $0^\circ \leq x \leq 120^\circ$ by $g(x) = 2 + 4 \cos 6x$.

(a) On the axes, sketch the graph of $y = g(x)$.

[3]

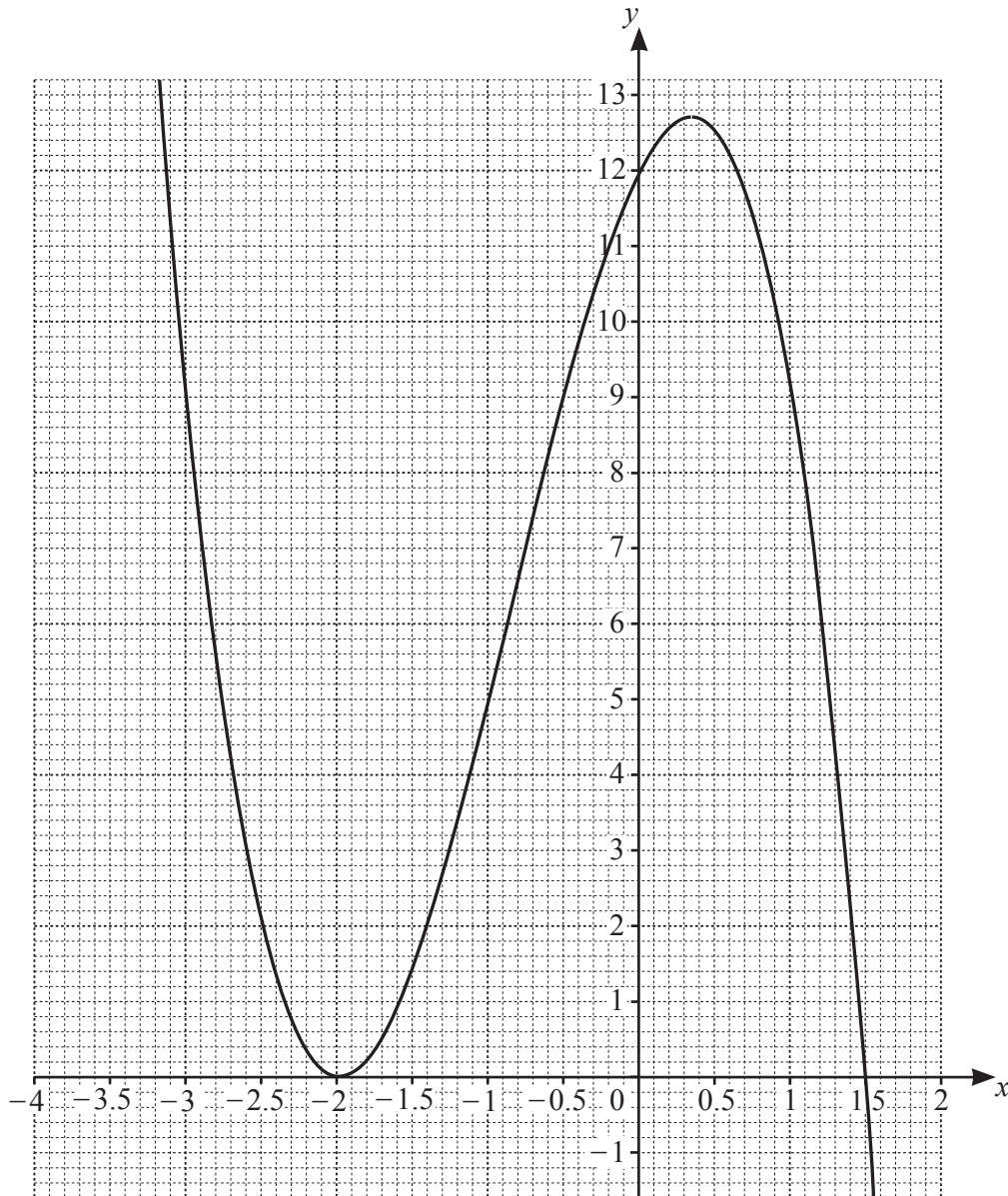


(b) State the amplitude of g .

[1]

(c) State the period of g .

[1]



The diagram shows the graph of $y = h(x)$ where $h(x) = (x+a)^2(b+cx)$ and a , b and c are integers. The curve meets the x -axis at the points $(-2, 0)$ and $(1.5, 0)$ and the y -axis at the point $(0, 12)$.

(a) Find the values of a , b and c .

[2]

(b) Use the graph to solve the inequality $h(x) \leq 9$.

[3]

- 4 (a) Solve the equation $5^{2y-1} = 6 \times 3^y$, giving your answer correct to 3 decimal places. [3]

- (b) Solve the equation $e^{2x} - 4 + 3e^{-2x} = 0$, giving your answers in exact form. [4]

- 5 The volume, V , of a sphere of radius r is given by $V = \frac{4}{3}\pi r^3$.

The volume of a sphere is increasing at a constant rate of $24 \text{ cm}^3 \text{ s}^{-1}$. Find the rate of increase of the radius when the radius is 6 cm. [4]

- 6 (a) The position vectors of the points P , Q and R relative to an origin O are $\begin{pmatrix} 4 \\ 7 \end{pmatrix}$, $\begin{pmatrix} 8 \\ 5 \end{pmatrix}$ and $\begin{pmatrix} x \\ y \end{pmatrix}$ respectively. The point R lies on PQ extended such that $3\overrightarrow{QR} = 2\overrightarrow{PR}$. Use a vector method to find the values of x and y . [3]

- (b) You are given that $\mathbf{i}$ is a unit vector due east and $\mathbf{j}$ is a unit vector due north.

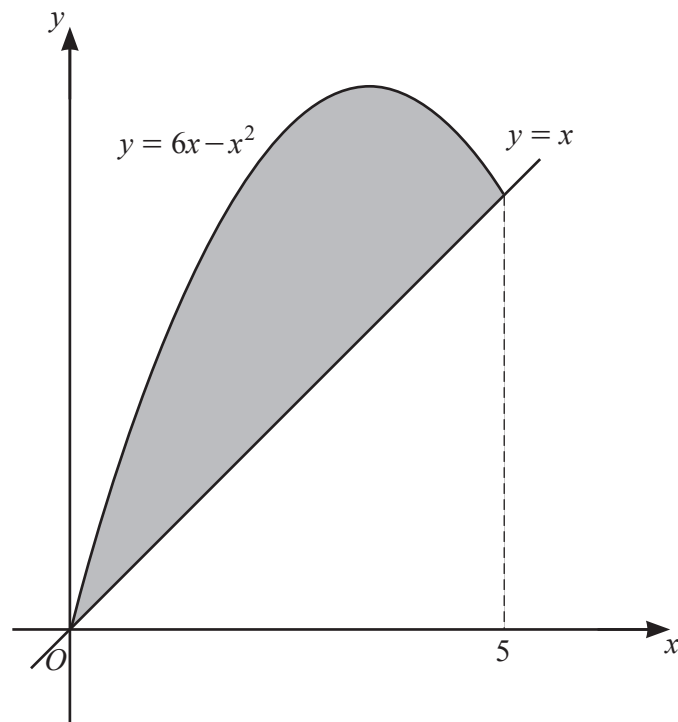
Three vectors, $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are in the same horizontal plane as $\mathbf{i}$ and $\mathbf{j}$ and are such that $\mathbf{a} + \mathbf{b} = \mathbf{c}$.
The magnitude and bearing of $\mathbf{a}$ are 5 and 210° .
The magnitude and bearing of $\mathbf{c}$ are 10 and 330° .

- (i) Find $\mathbf{a}$ and $\mathbf{c}$ in terms of $\mathbf{i}$ and $\mathbf{j}$. [2]

(ii) Find the magnitude and bearing of **b**.

[5]

7 (a)

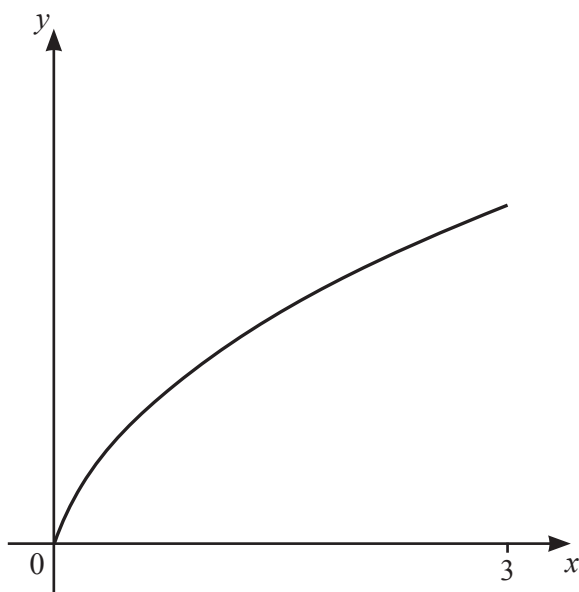


The diagram shows the curve $y = 6x - x^2$ for $0 \leq x \leq 5$ and the line $y = x$. Find the area of the shaded region. [4]

(b) (i) Find $\int \left(\frac{1}{(2x-6)^3} + \cos x \right) dx$. [3]

(ii) Find $\int \frac{(x^4+1)^2}{2x} dx$. [3]

8 (a)



The diagram shows the graph of $y = f(x)$ where f is defined by $f(x) = \frac{3x}{\sqrt{5x+1}}$ for $0 \leq x \leq 3$.

(i) Given that f is a one-one function, find the domain and range of f^{-1} . [3]

(ii) Solve the equation $f(x) = x$. [2]

(iii) On the diagram above, sketch the graph of $y = f^{-1}(x)$. [2]

(b) The functions g and h are defined by

$$g(x) = \sqrt[3]{8x^3 + 3} \quad \text{for } x \geq 1,$$

$$h(x) = e^{4x} \quad \text{for } x \geq k.$$

(i) Find an expression for $g^{-1}(x)$. [2]

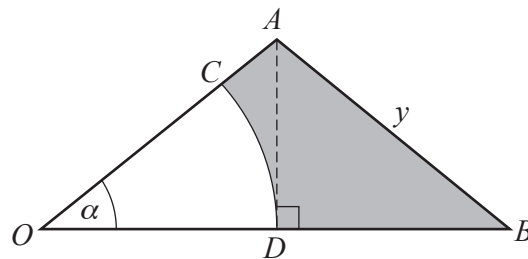
(ii) State the least value of the constant k such that $gh(x)$ can be formed. [1]

(iii) Find and simplify an expression for $gh(x)$. [1]

9 In this question all lengths are in centimetres and all angles are in radians.

(a) The area of a sector of a circle of radius 24 is 432 cm^2 . Find the length of the arc of the sector. [4]

(b)



The diagram shows an isosceles triangle, OAB , with $AO = AB = y$ and height AD . OCD is a sector of the circle with centre O . Angle AOB is α .

(i) Find an expression for OB in terms of y and α . [1]

(ii) Hence show that the area of the shaded region can be written as $\frac{y^2}{2} \cos \alpha (2 \sin \alpha - \alpha \cos \alpha)$. [3]

- 10** In the expansion of $\left(ax + \frac{b}{x^2}\right)^9$, where a and b are constants with $a > 0$, the term independent of x is $-145\,152$ and the coefficient of x^6 is -6912 . Show that $a^2b = -12$ and find the value of a and the value of b . [7]

Question 11 is printed on the next page.

- 11 The line with equation $x + 3y = k$, where k is a positive constant, is a tangent to the curve with equation $x^2 + y^2 + 2y - 9 = 0$. Find the value of k and hence find the coordinates of the point where the line touches the curve. [9]

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