Cambridge Assessment International Education

# Cambridge O Level

| CANDIDATE NAME | |
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| CENTRE NUMBER | | | | | | CANDIDATE NUMBER | | | | |
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*8611364537*

**ADDITIONAL MATHEMATICS** **4037/12**

Paper 1 **May/June 2024**

**2 hours**

You must answer on the question paper.

No additional materials are needed.

**INSTRUCTIONS**

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

**INFORMATION**

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages.

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## *Mathematical Formulae*

### 1. ALGEBRA

*Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

*Binomial Theorem*

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where $n$ is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

*Arithmetic series* $\quad u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

*Geometric series* $\quad u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

### 2. TRIGONOMETRY

*Identities*

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

*Formulae for $\Delta ABC$*

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

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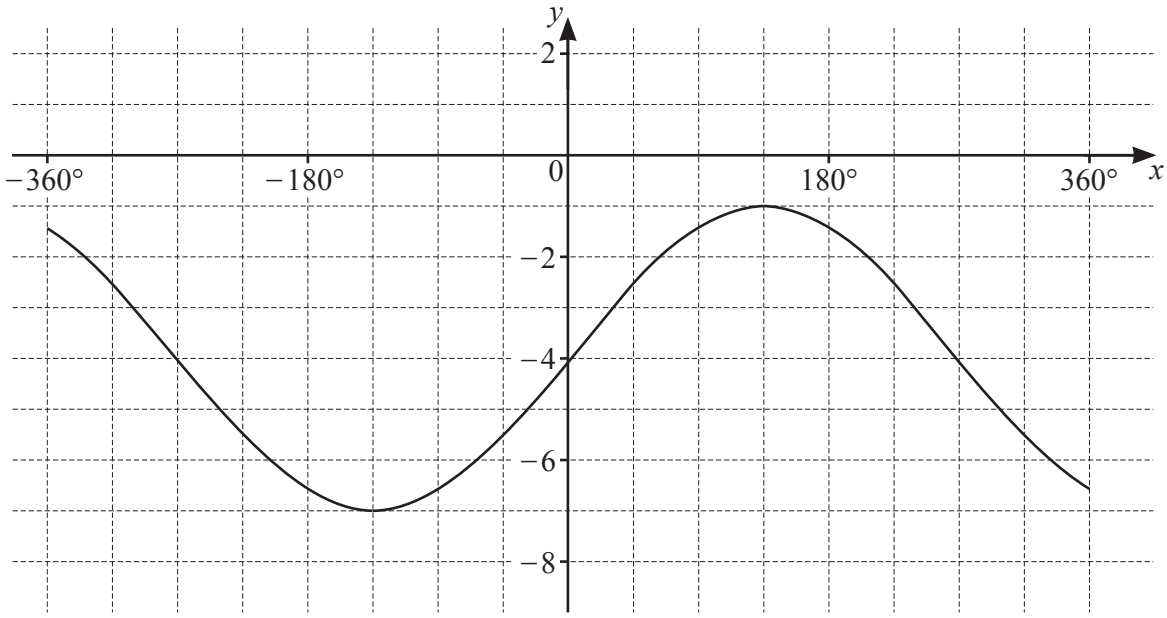
**1**




The diagram shows the graph of $y = a\sin bx + c$ for $-360° \leqslant x \leqslant 360°$, where $a$, $b$ and $c$ are constants. Find the values of $a$, $b$ and $c$. [3]

**2** Given that $\log_3 r + 2\log_9 s = 8$, find the value of $rs$. [3]

**3** Given that $y = \tan\frac{x}{2}$, find the exact value of $\frac{dy}{dx}$ when $x = \frac{\pi}{3}$. [4]

**4** A team of 8 people is to be formed from 6 teachers, 5 doctors and 4 police officers.

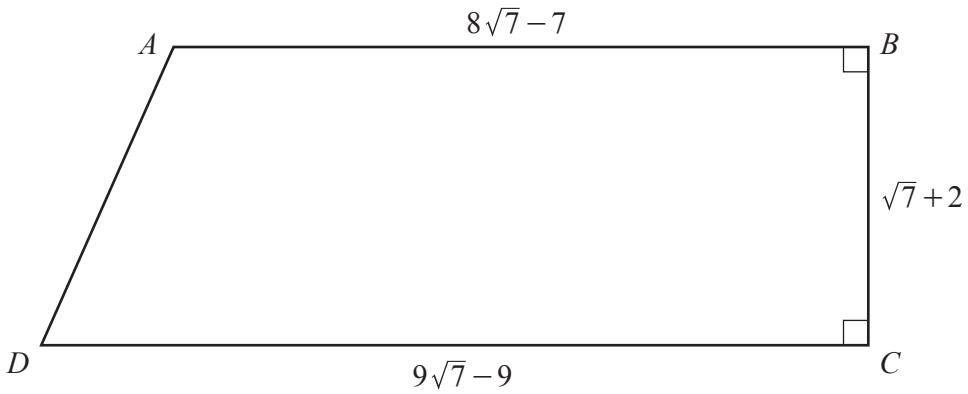
**(a)** Find the number of teams that can be formed. [1]

**(b)** Find the number of teams that can be formed without any teachers. [1]

**(c)** Find the number of teams that can be formed with the same number of doctors as teachers. [4]

**5 DO NOT USE A CALCULATOR IN THIS QUESTION.**

In this question, all lengths are in centimetres.



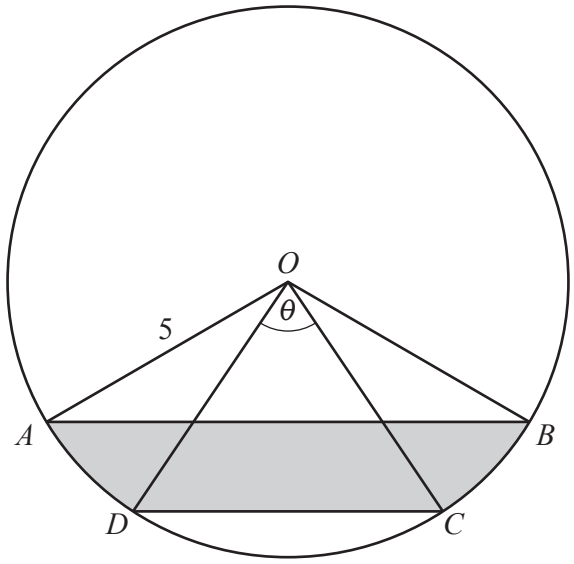

The diagram shows the trapezium *ABCD*. The lengths of *AB*, *BC* and *CD* are $8\sqrt{7}-7$, $\sqrt{7}+2$ and $9\sqrt{7}-9$ respectively. The line *BC* is perpendicular to the lines *AB* and *CD*.

**(a)** Find the perimeter of the trapezium, giving your answer in its simplest form. [3]

**(b)** Find the area of the trapezium, giving your answer in the form $p\sqrt{7}+q$, where $p$ and $q$ are rational numbers. [3]

**(c)** Find $\cot DBC$, giving your answer in the form $r\sqrt{7}+s$, where $r$ and $s$ are simplified rational numbers. [3]

**6** In this question, all lengths are in metres and all angles are in radians.




The diagram shows a circle with centre $O$ and radius 5. The points $A$, $B$, $C$ and $D$ lie on the circumference of the circle. Angle $DOC = \theta$. Angle $AOD$ = angle $COB = 0.5$. The length of the minor arc $DC$ is 3.75.

**(a)** Show that $\theta = 0.75$. [1]

**(b)** Find the perimeter of the shaded region. [5]

**(c)** Find the area of the shaded region. [3]

**7** **(a)** The line $y = 3x - 2$ intersects the curve $2x^2 - xy + y^2 = 2$ at the points $A$ and $B$. The point $C$ with coordinates $\left(k, \frac{7}{8}\right)$ lies on the perpendicular bisector of the line $AB$. Find the exact value of $k$.

[9]

**(b)** The point $D$ lies on the perpendicular bisector of $AB$ such that $D$ is a reflection of $C$ in the line $AB$. Find the coordinates of $D$. [2]

**8** A curve has equation $y = \dfrac{(3x^2 - 5)^{\frac{1}{3}}}{x+4}$.

**(a)** Show that $\dfrac{dy}{dx}$ can be written in the form $\dfrac{Ax^2 + Bx + C}{(3x^2 - 5)^{\frac{2}{3}}(x+4)^2}$, where $A$, $B$ and $C$ are integers. [5]

**(b)** Hence find the $x$-coordinates of the stationary points on the curve. Give your answers in their simplest exact form. [3]

**9** In this question, all distances are in metres and time, $t$, is in seconds.

A particle $P$ moves with a speed of 14.5 parallel to the vector $\begin{pmatrix} -20 \\ 21 \end{pmatrix}$.

**(a)** Find the velocity vector of $P$. [2]

Initially, $P$ has position vector $\begin{pmatrix} 3 \\ 5 \end{pmatrix}$.

**(b)** Write down the position vector of $P$ at time $t$. [2]

A second particle $Q$ has position vector $\begin{pmatrix} -1 \\ 3 \end{pmatrix} + \begin{pmatrix} -5 \\ 7.5 \end{pmatrix} t$ at time $t$.

**(c)** Find, in terms of $t$, the distance between $P$ and $Q$ at time $t$. Simplify your answer. [4]

**(d)** Hence show that $P$ and $Q$ never collide. [2]

**10** **(a)** The first 3 terms of an arithmetic progression are $3\sin 2x$, $5\sin 2x$, $7\sin 2x$.

**(i)** Show that the sum to $n$ terms of this arithmetic progression can be written in the form $n(n+a)\sin 2x$, where $a$ is a constant. [3]

**(ii)** Given that $x = \frac{2\pi}{3}$, find the exact sum of the first 20 terms. [2]

**(b)** The first 3 terms of a geometric progression are $\ln 2y$, $\ln 4y^2$, $\ln 16y^4$.

**(i)** Find the $n$th term of this geometric progression. [2]

**(ii)** Find the sum to $n$ terms of this geometric progression, giving your answer in its simplest form. [2]

**(c)** The first 3 terms of a different geometric progression are $\left(2w-\frac{1}{4}\right)$, $\left(2w-\frac{1}{4}\right)^2$, $\left(2w-\frac{1}{4}\right)^3$.

Find the values of $w$ for which this geometric progression has a sum to infinity. [3]

**Question 11 is printed on the next page.**

**11** **(a)** Given that $y = x^2 \ln x$, find $\frac{dy}{dx}$. [2]

**(b)** Hence find $\int x \ln x \, dx$. [3]

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